HS Aschaffenburg
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# Beispiel zum BHC-Code mit $t_{Kor}$ = 3 im Galoisfeld GF($2^4$)

**n = 15**
**m = 10**
**k = 5**

**Generatorpolynom** $g(x)$ = ( 1 0 0 1 1 ) · ( 1 1 1 1 1 ) · ( 1 1 1 )

= 1 0 1 0 0 1 1 0 1 1 1

$u(x)$ = 1 0 1 1 1

$u(x) \cdot x^{10}$ = 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0

$v(x) = u(x) \cdot x^{10} + (u(x) \cdot x^{10}) \cdot \mathrm{MOD}\ g(x)$

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $u(x) \cdot x^{10} \cdot \mathrm{MOD}\ g(x)$ = | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | | | | |
| | | | | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | |
| | | | | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | |
| | | | | | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| | | | | | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| Rest = | | | | | | | | | | 1 | 0 | 1 | 0 | 0 | 1 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $v(x) = u(x) \cdot x^{10}$ + Rest = | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | | | | 1 | 0 | 1 | 0 | 0 | 1 |
| | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Fehler** $e(x) = x^{11} + x^6 + x^2$ = | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |

**Empfangswort** $w(x) = v(x) + e(x)$

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| = | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| | | | | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| | 1 | 0 | 1 | $\boxed{0}$ | 1 | 0 | 0 | 0 | $\boxed{1}$ | 1 | 0 | 1 | $\boxed{1}$ | 0 | 1 |

**Syndrome**

$$s_1 = w(\beta) = \beta^{14} + \beta^{12} + \beta^{10} + \beta^{6} + \beta^{5} + \beta^{3} + \beta^{2} + \beta^{0}$$

$$= \underline{\beta^{5}}$$

$$s_3 = w(\beta^3) = \beta^{42} + \beta^{36} + \beta^{30} + \beta^{18} + \beta^{15} + \beta^{9} + \beta^{6} + \beta^{0}$$

$$= \beta^{12} + \beta^{6} + \beta^{0} + \beta^{3} + \beta^{0} + \beta^{9} + \beta^{6} + \beta^{0}$$

$$= \underline{\beta^{6}}$$

$$s_5 = w(\beta^5) = \beta^{70} + \beta^{60} + \beta^{50} + \beta^{30} + \beta^{25} + \beta^{15} + \beta^{10} + \beta^{0}$$

$$= \beta^{10} + \beta^{0} + \beta^{5} + \beta^{0} + \beta^{10} + \beta^{0} + \beta^{10} + \beta^{0}$$

$$= \underline{\beta^{0}}$$

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## Fehlergleichungen:

| | | |
|---|---|---|
| $e(\beta) = \beta^{i} + \beta^{j} + \beta^{k} = s_1 =$ | $\underline{\beta^5}$ | System von |
| $e(\beta^3) = \beta^{3i} + \beta^{3j} + \beta^{3k} = s_3 =$ | $\underline{\beta^6}$ | 3 Gleichungen |
| $e(\beta^5) = \beta^{5i} + \beta^{5j} + \beta^{5k} = s_5 =$ | $\underline{\beta^0}$ | mit den 3 Unbekannten i, j, k |

## Lösung des Gleichungssystems:

zum Beispiel durch systematisches

Durchprobieren aller "15 über 3" = 455

Kombinationen der i = 0, 1, 2, 3, .... , 14
j = 0, 1, 2, 3, .... , 14
k= 0, 1, 2, 3, .... , 14

Dies wäre ein einfacher, nicht effektiver, aber

dennoch vollständiger Weg zu einer Lösung

Die Kombination i=11, j = 6, k= 2 ergibt z. B.:

$$e(\beta) = \beta^{11} + \beta^{6} + \beta^{2} = \beta^{11} + \beta^{6} + \beta^{2} = \beta^3 + \beta^2 + \beta + \beta^3 + \beta^2 + \beta^2$$

$$= \beta^2 + \beta = \beta^5 = \mathbf{s_1}$$

$$e(\beta^3) = \beta^{3\times 11} + \beta^{3\times 6} + \beta^{3\times 2} = \beta^3 + \beta^3 + \beta^6 = \beta^6 = \mathbf{s_3}$$

$$e(\beta^5) = \beta^{5\times 11} + \beta^{5\times 6} + \beta^{5\times 2} = \beta^{10} + \beta^0 + \beta^{10} = \beta^0 = \mathbf{s_5}$$

Daher ist diese Kombination die gesuchte Lösung, die 3 Fehler sitzen von rechts aus gezählt

an den Bitpositionen 12, 7 und 3 und können nun korrigiert werden.

Rechentechnisch **eleganter** ist die algebraische Lösung des oben angegebenen

Gleichungssystems, siehe **Dankmeier, "Grundkurs Codierung"**, 3. Auflage, 2006,

**Unterkapitel 3.7.7**, Seite 133 ff.und **Unterkapitel 3.8.3**, Seite 182 ff.